Rat Tumor Data: Two Models in STAN

Model 1: Beta-Binomial Model

$$
\begin{aligned}
y_i \mid \theta_i &\sim \text{Binomial}(n_i, \theta_i),\ \ i = 1, \ldots, k \\
\theta_i &\sim \pi(\theta) = \text{Beta}(\alpha, \beta) \\
\alpha &\sim \text{Normal}(2.4, 5) \\
\beta &\sim \text{Normal}(14, 5)
\end{aligned}
$$

Using `rstan`, we successfully ran the model. The `R` code and data are attached in the script `RatTumorExample.R`.

Model 2: DS(G, m), where $G = $ Beta(α, β)

Following Mukhopadhyay and Fletcher, 2018 (`https://www.nature.com/articles/s41598-018-28130-5`), we propose the following simple model that provides non-parametric inference with parametric computational cost. Let g be the density (pdf) of Beta(α, β), with G as its distribution function (cdf):

$$
\begin{aligned}
y_i \mid \theta_i &\sim \text{Binomial}(n_i, \theta_i),\ \ i = 1, \ldots, k \\
\theta_i &\sim \pi(\theta) \equiv \text{DS}(G, m) = g(\theta)\Big[1 + \sum_{j=1}^{m} c_j \text{Leg}_j(G(\theta))\Big] \\
\alpha &\sim \text{Normal}(2.4, 5) \\
\beta &\sim \text{Normal}(14, 5) \\
c_1, c_2, \ldots, c_m &\sim \text{Normal}(0, 5)
\end{aligned}
$$

where $\text{Leg}_j(u)$ is the jth orthonormalized shifted Legendre polynomial i.e.,

$$
\begin{aligned}
\text{Leg}_1(u) &= \sqrt{12}(u - \frac{1}{2}) \\
\text{Leg}_2(u) &= \sqrt{5}(6u^2 - 6u + 1) \\
&\vdots
\end{aligned}
$$

In `R` language, the density (pdf) of the DS(G, m) prior distribution is:

$$
\pi(\theta) = \texttt{dbeta}(\theta; \alpha, \beta) \left[1 + \sum_{j=1}^{m} c_j \underbrace{\text{Leg}_j(\texttt{pbeta}(\theta; \alpha, \beta))}_{\texttt{R}\text{ script }\texttt{LPbasis.R}\text{ is attached}} \right]
$$

OBJECTIVE: Our goal is to implement the DS(G, m) prior distribution as a custom prior in `STAN`. Once implemented, we can test DS(G, m) under MCMC. We appreciate any assistance you can provide to convert the prior into `STAN` language.